Distributed Relay Beamforming for Fairness-Aware Amplify-and-Forward Relaying under Correlated Relays Noise

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***Abstract*—This paper presents a distributed replay beamforming design for two-way relay networks (TWRNs) under correlated relay noise. This network consists of two transceivers end nodes and N parallel amplify-and-forward (AF) relay nodes, where noise at the AF relay nodes may be correlated due to common interference or propagation through multiple hops. This paper assumes a practical scenario with correlated relay noises in contrast to the majority of the related work that assumes independent relays noise. Our objective is to maximize the worst signal-to-noise power ratio (SNR) of the two end nodes under individual relay power constraints and total relay power constraint in order to improve the users fairness for the two end nodes. This relay beamforming design is a non-convex problem taking the form of max-min optimization which could be solved by decomposing it into a series of solvable sub-problems using bisection search. Semi-definite relaxation problem (SDR) is used for the reformulation of the problem. The effect of the noise correlation on the beamformer design is analyzed for two cases, i.e., the first case when a complete knowledge of the correlation structure, represented by the relay noise covariance matrix (K), is available to the relay and the second one when no knowledge is available to the relay. Simulation results show performance improvement in terms of users fairness in comparison with un-optimized AF relaying in both cases. The obtained result indicates that the knowledge of the correlation structure increases the system performance w.r.t the case where no knowledge is available.**

Keywords*—*Distributed Relay Beamforming, Fairness, Amplify-and-Forward, Max-min, Correlated Noise.

I. Introduction

Recently, the design of two-way relay networks (TWRN) has attracted several intensive researches [1–3]. In such networks, the relay node/nodes establish two-way communication between two or more transceivers. These nodes have either a single antenna or multi-antennas. There are many cooperative relaying schemes that have been proposed for the TWRN, such as amplify-and-forward (AF) [1], decode-and-forward (DF) [4], compress-and-forward (CF) [5]. Most of the practical relay-based systems adopt AF protocol with half-duplex (HD) nodes due to its simplicity.

Relay beamforming is an AF technique which adjusts complex amplification factors of all relays, known as relay beamforming weights, to satisfy a certain performance criterion subject to the available resources and/or quality-of-service (QoS) constraints. Multi-antenna relay beamforming has been explored to achieve higher spatial diversity [6, 7]. In communication nodes with size and hardware limitations, each node could only have a single antenna. In order to benefit from the multi-antenna gain in such nodes, a strategy known as distributed relay beamforming has been developed where the relaying nodes cooperate to produce a beam towards the destination [8, 9].

The main idea in relay beamforming schemes is to optimize the relay beamforming weights. Two different optimality criteria are used in [8] to design beamformers for TWRN, the first criteria is to minimize the total transmit power under QoS constraints. The second one is the max-min fairness design which maximizes the minimum signal-to-noise power ratio (SNR) under a total transmit power constraint. The max-min criteria have been drawing attention in the last decade as a result of the enormous growth of traffic in telecommunication networks that has been observed. The distribution of this traffic in the networks changes quickly, both in long and short time, and is therefore very difficult to predict.

One significant performance metric for a wireless network is the average throughput perceived by users. However, this metric alone is not adequate to judge the performance in a multi-user scenario. Generally, it is better to design the network such that resources are not dedicated to few users while others are left unserved, even if this leads to the maximization of the total network throughput [10]. The users fairness objective is to guarantee a fair access to the communication network [11]. However, increasing system throughput and fair access assurance are conflicting performance metrics in limited radio resources, since maximizing the total system throughput may be achieved in starving users with weak channel conditions. On the other hand, maintaining perfect fairness may result in significant degradation of total system efficiency which leads to searching for a tradeoff between these two metrics [12, 13]. Max-min fairness is considered to balance these two conflicting objectives[14, 15] as it prevents starving of any user and at the same time increases the data rate as much as possible. In this paper, we consider fairness as a crucial parameter to evaluate the network performance in a target wireless communication system.

In practical scenarios of AF beamforming, there are two issues to be taken into consideration, the noise correlation and the channel uncertainty. The noise between the nodes may be correlated due to several reasons such as common interference and noise propagation [16, 17] and the channel state information (CSI) could be imperfect for many reasons such as inaccurate channel estimation, quantization errors, feedback delay and errors, etc. [18, 19]. To improve achievable system performance in practical scenarios, it is important to design AF relay nodes under assumptions of correlated relay noise and imperfect CSI.

System designs in presence of correlated noise are not deeply studied for AF beamforming scenarios. The pioneer work in [16] presented a closed-form solution for maximizing the sum-rate for one-way AF relaying under total relay power constraint. The work in [17] assumes two-way relaying and its objective is to maximize one of the two SNRs under the constraint that the other one is greater than certain threshold. In [17], it is assumed that all network parameters are real-valued for simplicity and the noise at the relays is correlated. In [18], a robust AF beamforming design based on a max-min fairness approach was presented under assumptions of imperfect CSI and independent relay noise, where the minimum of the two end nodes SNRs is maximized under a total relays power constraint in presence of imperfect CSI. The formulation of the max-min optimization problem in [18] is a non-convex problem that is converted to a series of solvable sub-problems by the bisection search algorithm where each sub-problem is a relays power minimization in the form of semi-definite relaxation problem (SDR).

This paper presents a distributed AF relay beamforming design to improve the fair access for the network users in presence of noise correlations between relay nodes. In this context, a max-min fair design approach is used to optimally determine the relays distributed beamforming weights through the maximization of the worst SNRs at two transceivers under total relay nodes power constraint and considering extra N constraints for individual relay nodes power. Although this paper employs the max-min fairness approach like [18] for the same AF beamforming model, correlated relay noise is taken into consideration under the assumption of perfect CSI in contrast to [18]. More specially, the formulation of the max-min relay beamforming design is a non-convex optimization problem also like that in [18] but in this paper it could be solved by decomposing it into a series of solvable sub-problems using bisection search where each sub-problem is a feasibility check problem in the form of SDR problem. In addition, in contract to [17], the system model assumes a general complex-valued network parameters in contrast to the simplified two-way system model in [17].

The main contributions of this paper and the relation to related work are:

- Analyzing the effect of the noise correlation on the beamformer design under two scenarios, i.e., when a complete knowledge of the correlation structure is available to the relay and when this knowledge is not available to the relays.
- Jain's fairness index [20] is used as a metric for comparing the fairness of the two scenarios w.r.t a reference un-optimized beamforming of equal beamformer vector values which is not studied in [18].
- In addition to the total relay power constraint in [18], N extra individual relay power constraint are used to address the possibility of different energy constraints for the relay nodes involved in the beamforming process.
- The optimization problem is solved using bisection search that solves a series of solvable feasibility check problems in the form of SDR problem which is similar to the technique used in [18]. However, the feasibility check problems replaces the relays power minimization problems solved in each bisection iteration in [18] because of the difference in assumptions about the CSI certainty.

The rest of this paper is organized as follows: Section II describes the system models for the AF interference-limited TWRN, followed by the analysis of the distributed relay beamforming optimization problem in section III, Section IV is for simulation parameters and results and finally the conclusion in section V.

Notations: Boldface uppercase letters denote matrices, boldface lowercase letters denotes column vectors, $\odot$ denotes the Hadamard element-by-element product. $Tr(\mathbf{A})$ is the trace of the matrix $\mathbf{A}$. $diag(a)$ is a diagonal matrix with the vector a being its diagonal entries. Superscripts $(.)^T$,$(.)^*$, $(.)^H$ denotes the transpose, conjugate, and the hermitian respectively. $E(.)$ denotes the expectation. The distribution of a circular symmetric complex gaussian vector with mean μ and covariance matrix $\mathbf{K}$ is denoted by $\mathcal{CN}(\mu, \mathbf{K})$

II. System Model

In this paper, we consider a TWRN where two end-nodes exchange information via N relay nodes, as shown in Fig. 1. Each node is a single antenna device subject to half-duplex constraint where the nodes can not transmit and receive at the same time. All uplink and downlink channels vectors are modeled as $\mathcal{CN}(0, d_i^{-v})$ where d_i is the distance between the node and the i^{th} relay and v is the path loss exponent. In Fig.1, for i=1,2, the uplink channels vector from S_i to the N relay nodes is denoted as $\boldsymbol{f_i}$ where $\boldsymbol{f_i} = (f_{i1}, f_{i2}, ..., f_{iN})^T$ and the downlink channels vector from the relay nodes to S_i is denoted as $\boldsymbol{g_i}$ where $\boldsymbol{g_i} = (g_{i1}, g_{i2}, ..., g_{iN})^T$. s_i is the transmit signal from S_i with $E(|s_i|^2) = P_{Si}$, and $\boldsymbol{n_R} = (n_1, n_2, ..., n_N)^T$ is the relays correlated gaussian noise which is drawn according to $\mathcal{CN}(\mathbf{0}, \mathbf{K})$ where n_k is the k^{th} element of $\boldsymbol{n_R}$ and the covariance matrix given by $\mathbf{K} = E[\boldsymbol{n_R}\boldsymbol{n_R}^H]$.

The communication process uses multiple access broadcast (MABC) protocol where the transmission process takes place in two time slots. In the first times slot, known as multiple access (MAC) phase, both S_1 and S_2 transmit their signals to relay nodes simultaneously. Each relay node receive superimposed signal from both transceivers in addition to the relay noise.

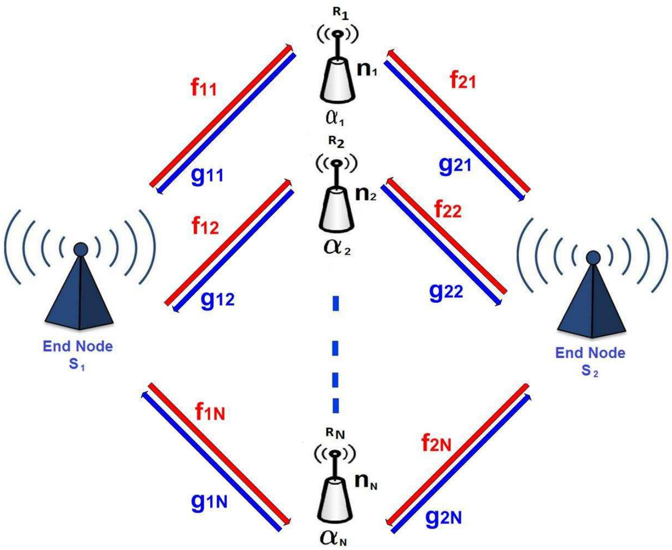

Fig. 1. Two-way relay channel using N parallel relay nodes and two end nodes for the AF beamforming scenarios.

The received signal at the k_{th} relay node can be written as:

$$y_{rk} = f_{1k}\, s_1 + f_{2k}\, s_2 + n_k \tag{1}$$

The received signals vector at all the relay nodes can be expressed as:

$$\boldsymbol{y_r} = \boldsymbol{f_1}\, s_1 + \boldsymbol{f_2}\, s_2 + \boldsymbol{n_R} \tag{2}$$

Each relay node multiplies its received signal with a complex amplification factor α_k to adjust its amplitude and phase and then broadcast the multiplication result to both transceivers. The transmitted signals vector from the N relay nodes can be written as in [18] as:

$$\boldsymbol{x_r} = \boldsymbol{\alpha} \odot \boldsymbol{y_r} = \boldsymbol{\alpha} \odot \boldsymbol{f_1}\, s_1 + \boldsymbol{\alpha} \odot \boldsymbol{f_2}\, s_2 + \boldsymbol{\alpha} \odot \boldsymbol{n_R} \tag{3}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, ..., \alpha_N)^T$ is the beamforming vector.

During the second phase of the AF, broadcast phase (BC), the received signals at each transceiver are given by:

$$\begin{aligned} y_{si} &= \sum_{k=1}^{N} \boldsymbol{g_i} \odot \boldsymbol{x_r} + n_{Si} \\ &= \sum_{k=1}^{N} (g_{ik}\, \alpha_k\, f_{ik}\, s_i + g_{ik}\, \alpha_k\, f_{jk}\, s_j + g_{ik}\, \alpha_k\, n_{Rk}) + n_{Si} \end{aligned} \tag{4}$$

where $j = 2$ if $i = 1$ and $j = 1$ if $i = 2$, $\boldsymbol{g_i}$ is independent from $\boldsymbol{f_i}$ as we assume a general scenario with no channels reciprocity, and n_{Si} are additive white gaussian noises (AWGN) at the destinations with variance σ_i^2. Further, we assume that there is no correlation between destinations noises n_{S1} and n_{S2} and the relay noise vector $\boldsymbol{n_R}$ as these noise processes occur at two different time slots.

Equation (4) can be reformulated as:

$$y_{si} = \boldsymbol{g_i}^T (\boldsymbol{\alpha} \odot \boldsymbol{f_i})\, s_i + \boldsymbol{g_i}^T (\boldsymbol{\alpha} \odot \boldsymbol{f_j})\, s_j + \boldsymbol{g_i}^T (\boldsymbol{\alpha} \odot \boldsymbol{n_R})\, s_i + n_{Si} \tag{5}$$

By assuming a perfect CSI for all nodes, a self-interference cancelation process could be performed on the received signals at both end nodes S_1 and S_2 from (5) and the resulting signal at both ends are given by:

$$\begin{aligned} \bar{y}_{si} &= \boldsymbol{g_i}^T (\boldsymbol{\alpha} \odot \boldsymbol{f_j})\, s_j + \boldsymbol{g_i}^T (\boldsymbol{\alpha} \odot \boldsymbol{n_R})\, s_i + n_{Si} \\ &= \boldsymbol{\alpha}^T \boldsymbol{G_i}\, \boldsymbol{f_j} + \boldsymbol{\alpha}^T \boldsymbol{G_i}\, \boldsymbol{n_R} + n_{Si} \end{aligned} \tag{6}$$

where $\boldsymbol{G_i} = diag(\boldsymbol{g_i})$ for i = 1, 2. The received SNR at end node S_i can be expresses as:

$$\begin{aligned} SNR_i &= P_{Si}\, \frac{|\boldsymbol{\alpha}^T \boldsymbol{G_i}\, \boldsymbol{f_j}|^2}{|\boldsymbol{\alpha}^T \boldsymbol{G_i}\, \boldsymbol{n_R}|^2 + \sigma_i^2} \\ &= P_{Si}\, \frac{\boldsymbol{\alpha}^H \boldsymbol{G_i}\, \boldsymbol{f_j}\, \boldsymbol{f_j}^H \boldsymbol{G_i}^H \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \boldsymbol{G_i}\, \boldsymbol{K}\, \boldsymbol{G_i}^H \boldsymbol{\alpha} + \sigma_i^2} \end{aligned} \tag{7}$$

III. Distributed Relay Beamforming Design

The objective is to design the beamformer vector weights of the available relay nodes in order to maximize the worst SNR of the two end nodes to improve the users fairness under total relay nodes power constraint and individual relay node power constraints. The total power constraint is used to ensure energy efficient design for the system. The individual power constraints on the relay nodes is to make sure that our design does not violate the specification of the relay nodes involved in the beamforming process. This is considered a max-min optimization problem which is a quasi-convex quadratic program [21–23]. This problem could be recasted as semi-definite program (SDP) with additional rank-1 constraints on the positive semi-definite matrix variables. The approach for the solutions is to use semi-definite relaxation (SDR) by dropping the rank-1 constraint.

A. *Optimization Problem*

The optimization problem can be stated as

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{maximize}} \quad & \min_{i=1,2} \; SNR_i \\ \text{subject to} \quad & \boldsymbol{\alpha}^H [P_{S1}\, |\boldsymbol{F_1}|^2 + P_{S2}\, |\boldsymbol{F_2}|^2 + \boldsymbol{K} \odot \boldsymbol{I}] \boldsymbol{\alpha} \leq P_{sum} \\ & \boldsymbol{\alpha}^H [P_{S1}\, f_{1,k}^2 + P_{S2}\, f_{2,k}^2 + \sigma_k^2] \boldsymbol{I_k} \boldsymbol{\alpha} \leq P_k, \\ & \quad k = 1 : N \end{aligned} \tag{8}$$

where the first constraint ensures the total relay nodes power does not exceed a certain threshold value P_{sum}, the second group of constraints are N constraints for the individual relay nodes to ensure that the k_{th} relay power does not exceed a threshold value of P_k, $\boldsymbol{F_i} = diag(\boldsymbol{f_i})$, $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{I_k} = diag(0, ..., 0, 1, 0, ..., 0)$ is a diagonal matrix with the k_{th} element equal to 1, and σ_k^2 is the k_{th} diagonal element of $\boldsymbol{K}$. The above optimization problem differs from the one in [18] in that it does not only account for the total power constraint but also, the individual relay power constraint which is very useful for battery-operated relay nodes.

Problem (8) could be reformulated as:

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{maximize}} \quad & \min_{i=1,2} \; \frac{\boldsymbol{\alpha}^H \boldsymbol{R_i} \boldsymbol{\alpha}}{\boldsymbol{\alpha}^H \boldsymbol{M_i} \boldsymbol{\alpha} + 1} \\ \text{subject to} \quad & \boldsymbol{\alpha}^H \boldsymbol{Z} \boldsymbol{\alpha} \leq P_{sum} \\ & \boldsymbol{\alpha}^H \boldsymbol{C_k} \boldsymbol{\alpha} \leq P_k, \quad k = 1 : N \end{aligned} \tag{9}$$

where:

$$\begin{aligned}\boldsymbol{R_i} &= G_i \boldsymbol{f_j} \boldsymbol{f_j}^H G_i^H,\ \boldsymbol{M_i} = G_i K G_i^H / \sigma^2 \\ \boldsymbol{Z} &= P_{S1} |\boldsymbol{F_1}|^2 + P_{S2} |\boldsymbol{F_2}|^2 + K \odot I \\ \boldsymbol{C_k} &= [P_{S1}\, f_{1,k}^2 + P_{S2}\, f_{2,k}^2 + \sigma_k^2] I_k\end{aligned} \tag{10}$$

Assuming a semi-definite matrix $\boldsymbol{X} = \boldsymbol{\alpha}\boldsymbol{\alpha^H}$, the SNRs can be written as

$$SNR_i(\boldsymbol{X}) = \frac{\mathrm{Tr}(\boldsymbol{X}\boldsymbol{R_i})}{\mathrm{Tr}(\boldsymbol{X}\boldsymbol{M_i}) + 1} \tag{11}$$

This introduces a non-convex rank-1 constraint , $rank(\boldsymbol{X}) = 1$, to the optimization problem which could be removed by using a SDR approach. The new formulation could be written as:

$$\begin{aligned} &\underset{\boldsymbol{X}}{\text{maximize}} && \min_{i=1,2}\ SNR_i(\boldsymbol{X}) \\ &\text{subject to} && \mathrm{Tr}(\boldsymbol{X}\boldsymbol{Z}) \le P_{sum} \\ &&& \mathrm{Tr}(\boldsymbol{X}\boldsymbol{C_k}) \le P_k, \quad k = 1 : N \\ &&& \boldsymbol{X} \succeq 0. \end{aligned} \tag{12}$$

where $\boldsymbol{X} \succeq 0$ means that $\boldsymbol{X}$ is a positive semi-definite matrix.

B. Feasibility Check Problem

The standard method for solving problem (12) is by converting it to the following quasi-convex form where γ is an auxiliary variable:

$$\begin{aligned} &\text{maximize} && \gamma \\ &\text{subject to} && SNR_i(\boldsymbol{X}) \ge \gamma \\ &&& \mathrm{Tr}(\boldsymbol{X}\boldsymbol{Z}) \le P_{sum} \\ &&& \mathrm{Tr}(\boldsymbol{X}\boldsymbol{C_k}) \le P_k, \quad k = 1 : N \\ &&& X \succeq 0. \end{aligned} \tag{13}$$

The quasi-convex form in (13) can be solved by iterative processes such as the bisection search algorithm [24]. The bisection algorithm begins the search with an interval that is known to contain a solution and iteratively bisect the interval until reaching the solution. The global optimum value of $\gamma^\star$ can be found by solving the following convex feasibility check problem for each iteration of the auxiliary variable γ :

$$\begin{aligned} &\text{find} && \boldsymbol{X} \\ &\text{subject to} && SNR_i(\boldsymbol{X}) \ge \gamma \\ &&& \mathrm{Tr}(\boldsymbol{X}\boldsymbol{Z}) \le P_{sum} \\ &&& \mathrm{Tr}(\boldsymbol{X}\boldsymbol{C_k}) \le P_k, \quad k = 1 : N \\ &&& X \succeq 0. \end{aligned} \tag{14}$$

The feasibility check problem in (14) is a SDR problem that could be solved using optimization toolboxes [25]. The issue when using SDR is that the solution $\boldsymbol{X}^\star$ that achieves the optimum $\gamma^\star$ is the globally optimal to problem (13) but it is not the optimum for problem (8). The problem of converting a globally optimal solution $\boldsymbol{X}^\star$ to problem (13) into a feasible solution $\tilde{\boldsymbol{\alpha}}$ to problem (8) depends on the rank of $\boldsymbol{X}^\star$ which is not a rank-1 in general due to the relaxation process.

If $\boldsymbol{X}^\star$ is of rank-1, then its principal component is the optimal solution of the original problem (8) where $\boldsymbol{X}^\star = \boldsymbol{\alpha}^\star \boldsymbol{\alpha}^{\star T}$, and $\boldsymbol{\alpha}^\star$ is the principal component of $\boldsymbol{X}^\star$ [26]. Otherwise, if the rank of $\boldsymbol{X}^\star$ is more than 1, then we must extract from it a vector $\tilde{\alpha}$ that is feasible for problem (8). There are many ways to extract this such as eigenvector approximation or randomization [18, 27–29]. However, it must be emphasized that even though the extracted solution is a feasible solution for problem (8), it is in general a sub-optimal solution [27] and in this case, $\gamma^\star$ is the upper bound of problem (12) [29].

IV. SIMULATION RESULTS AND ANALYSIS

In this section, numerical results for the distributed beamformer vector design is presented for interference-limited TWRN. This network consists of N parallel relay nodes and two transceivers end nodes. The number of relays is assumed to be 5 nodes deployed randomly in the area between the two transceivers. The uplink and downlink channel coefficients between the two transceivers and the relays, $\boldsymbol{f_1}, \boldsymbol{f_2}, \boldsymbol{g_1}$, and $\boldsymbol{g_2}$ are modeled as $\mathcal{CN}(0, d^{-v})$, which denotes a circularly symmetric complex gaussian distribution with zero mean and d^{-v} variance, d is the distance between the relay node the transceiver node and v is the path loss exponent which is assumed to be 2.5 in this simulation. A complete CSI is assumed for all nodes.

In this paper, three beamforming scenarios are compared. The first scenario is the un-optimized AF relay beamforming where all the elements of the relay beamforming vector are assumed to be equal. The second scenario assumes that the relays has knowledge of the correlation structure represented by the relays noise covariance matrix (**K**). The last scenario assumes that the relays has no knowledge about the correlation structure. The relay beamforming design in this case assumes independent noises at the relay nodes which produces a diagonal noise covariance matrix given by $\mathbf{K} \odot \mathbf{I}$.

The objective is to improve the fair access for the network users under correlated relays noise. In this context, a max-min fair design approach given by (13) is compared to the un-optimized AF relay beamforming. Jain's fairness index [20] is used as a metric for the performance comparison which could be calculated as:

$$F_i = \frac{\sum_{i=1}^{M} r_i}{M \sum_{i=1}^{M} r_i^2} \tag{15}$$

where r_i is the achieved rate of the i^{th} user. The Jain's fairness index is a continuous fairness index ranges from 0 to 1 where the higher the index the higher the users fairness.

The solution of the optimization problem (13) is not a rank-1 in general and eigenvector approximation techniques is used in the simulation for extracting the feasible solution for the original problem (8). Let r be the rank of $\boldsymbol{X}^\star$ where the eigen-decomposition of $\boldsymbol{X}^\star$ is given by:

$$\boldsymbol{X}^\star = \sum_{j=1}^{r} \lambda_j b_j b_j^T \tag{16}$$

where $\lambda_1 \ge \lambda_2 \ge .... \ge \lambda_r \ge 0$ are the eigen values and $b_1, b_2, ..., b_r$ are their respective eigenvectors. The best rank-1

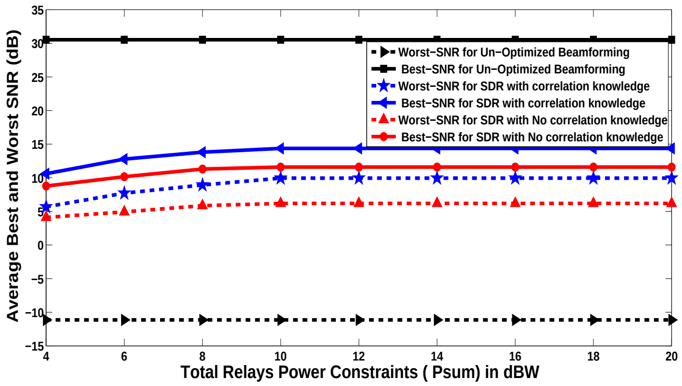

Fig. 2. Average achieved best and worst SNR versus the total relay power constraint for the AF beamforming with N parallel relay nodes.

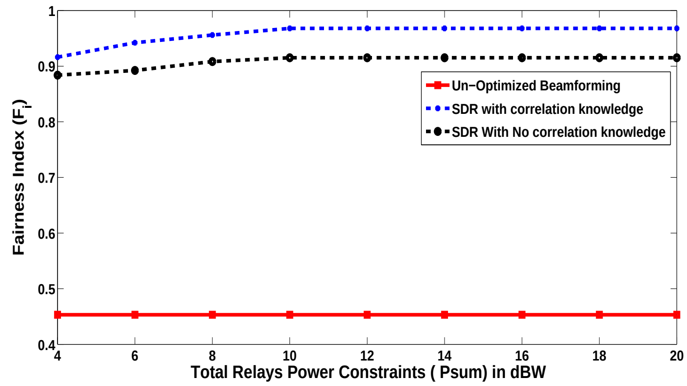

Fig. 3. Jain's fairness index versus the total relay power constraint for the AF beamforming with N parallel relay nodes.

approximation of $\boldsymbol{X}^\star$ is given by $\boldsymbol{X_1}^\star = \lambda_1 b_1 b_1^T$ [18, 27] and the approximate solution of problem (8), $\tilde{\boldsymbol{\alpha}}$, is given by:

$$\tilde{\boldsymbol{\alpha}} = \sqrt{\lambda_1}\, b_1 \tag{17}$$

In Fig. 2, the average value of the higher SNR of the two transceivers, i.e. best SNR, and the lower SNR, i.e. worst SNR and, are compared for the un-optimized beamforming case and the proposed beamformer design. It is shown that the proposed design improves the worst SNR for both cases with and without the complete relay knowledge of the correlation structure. This is a result of the objective of the optimization problem that maximizes the worst-SNR while this improvement is accompanied by a reduction in the best-SNR with a final result of improving the users' fairness by reducing the difference in rate between the two transceivers. Fig. 3 shows that the fairness index is increased by using the proposed design w.r.t the un-optimized beamforming which means an improvement in the users fairness as a result of reducing the difference in rate between the two transceivers by using the max-min design. The improvement in the case where the relays have knowledge about the correlation structure is more than that of the case with no knowledge about correlation structure. This could be considered as justification to any network overhead required to learn about the correlation.

V. Conclusion

In this paper, a fairness-aware distributed relay beamforming design under correlated relay noise was proposed for interference-limited TWRN. The proposed design is based on maximizing the worst achieved SNR of the two users which is a quasi-convex problem. Bisection search algorithm is used for solving a series of feasibility check problems in the form of SDR problems. The fairness index comparison of the proposed design to the un-optimized AF beamforming proves the validity of the proposed system to improve the users fairness. It also show that the knowledge of the correlation structure of noise between relays, represented by the relay noise covariance matrix, provides a higher improvement in the fairness index more than the case where no knowledge available to the relays. Our future work is to extend the proposed analysis to more realistic and generalized scenarios assuming both correlated relay noise and imperfect CSI in order to completely evaluate the fairness in such practical assumption of TWRNs.

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